Extreme values theory and landing distance

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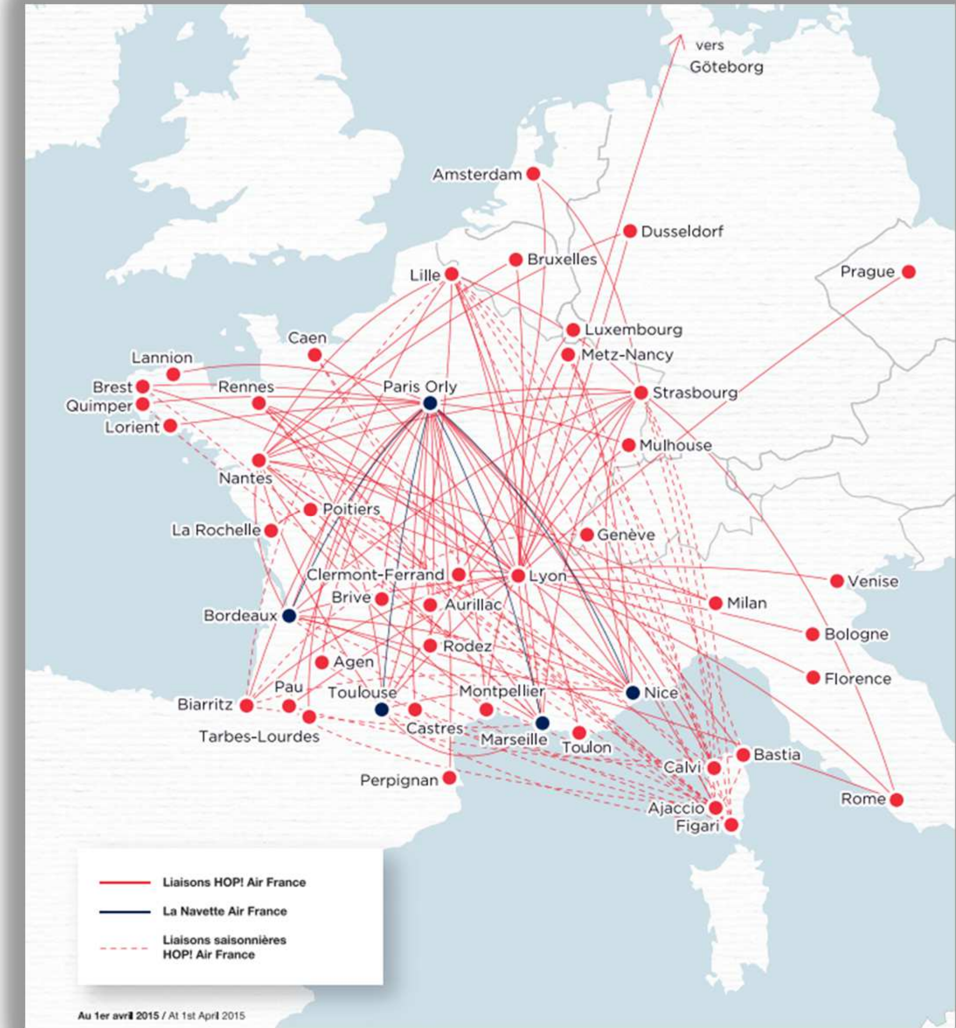
HOP ! Safety & Compliance Direction

Network and figures

Hop! Air France short-haul fleet



42 — **Embraer**
190
170
145

27 — **Bombardier**
CRJ 1000
CRJ 700

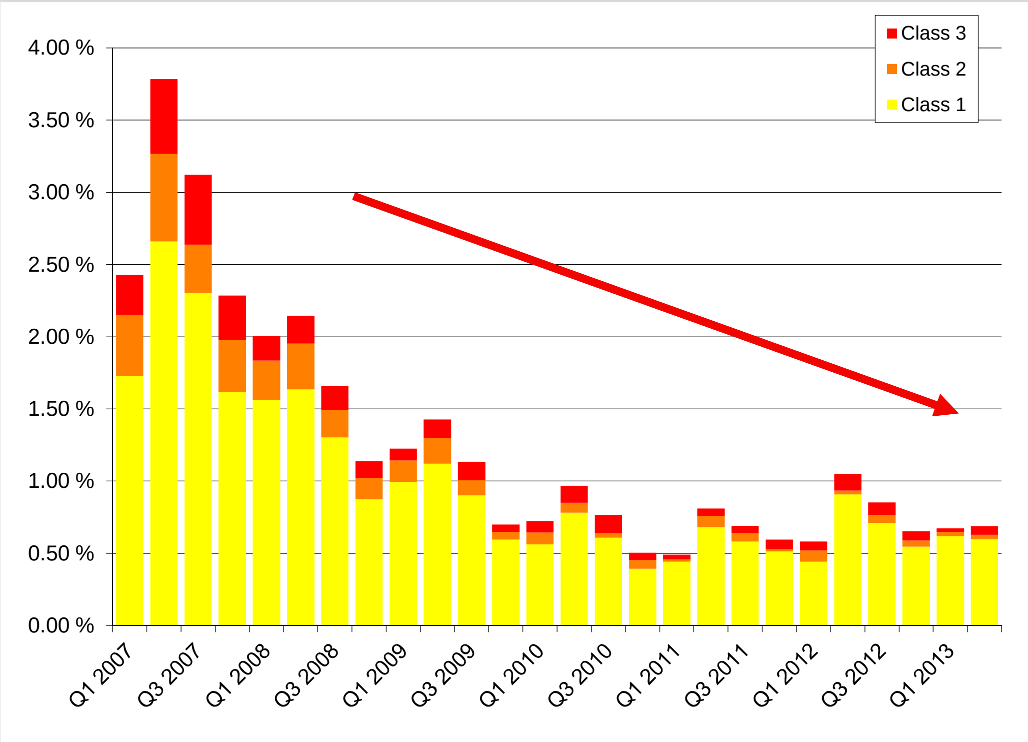
22 — **ATR**
ATR 72-500/600
ATR 42-500

Figures :
- 600 flights / day
- 91 aircrafts
- 50 stations
- 13 millions passengers
- 130 routes

FDM outputs, the basic approach

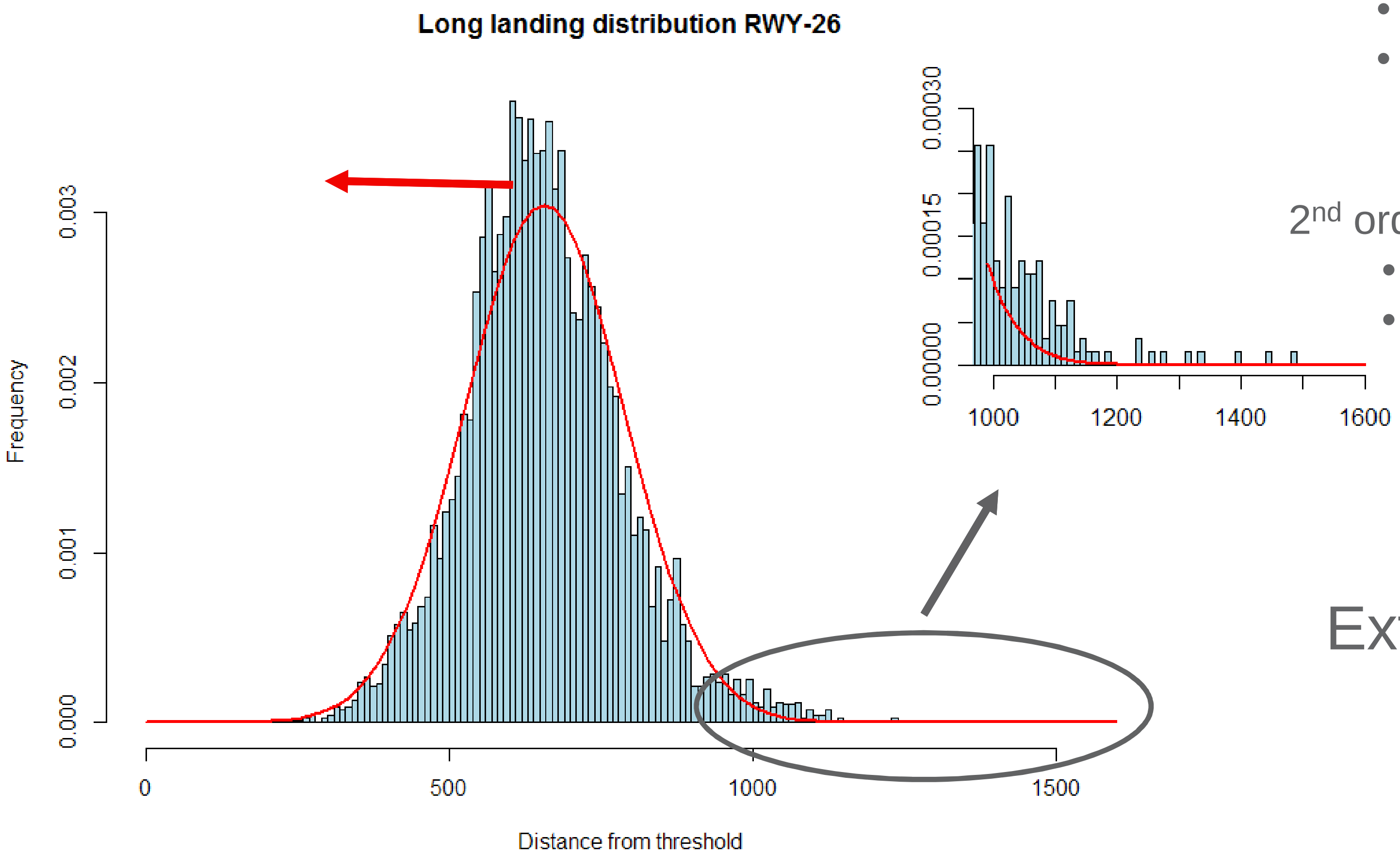
Event detection

Event classes



- How far are we from the class threshold ?
- What is the level of risk ?
- Is it the best way to measure the safety performance ?
- **Need for a model**

FDM outputs, another approach
Landing distance



Normal law (Gaussian)
- mean = 657 m
- sd = 131 m

2^{nd} order
- skewness = 0,53
- Excess kurtosis = 1,55

Extreme values

Risk management is the management of the unexpected events,

Extreme events are events with a very low probability of occurrence,

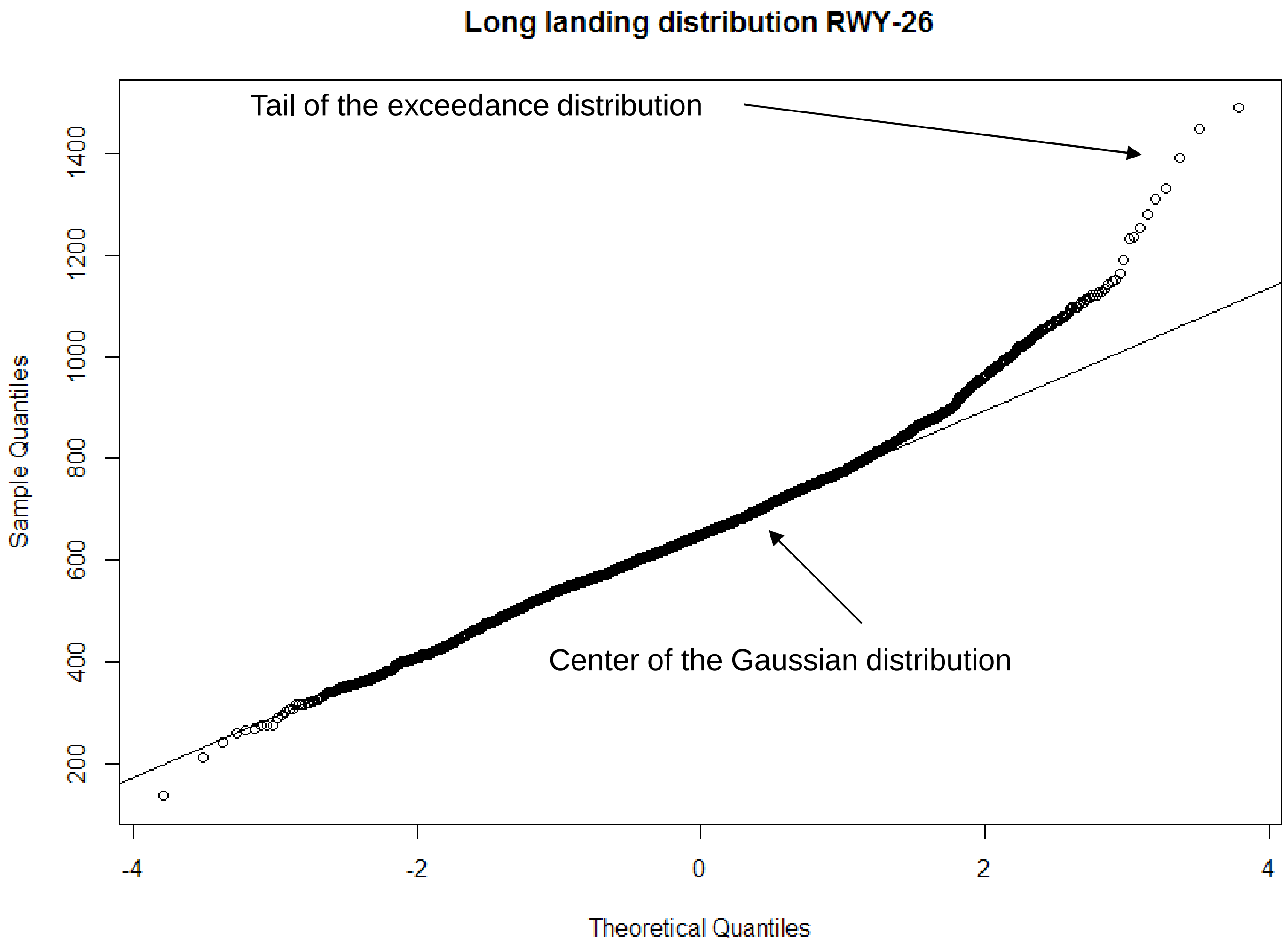
Normal distribution confers to a " predictable character “ (mean / sd),

Theory of the extreme values appears to be a more adapted tool to solve the problem.

Extreme value theory

Dataset characterisation

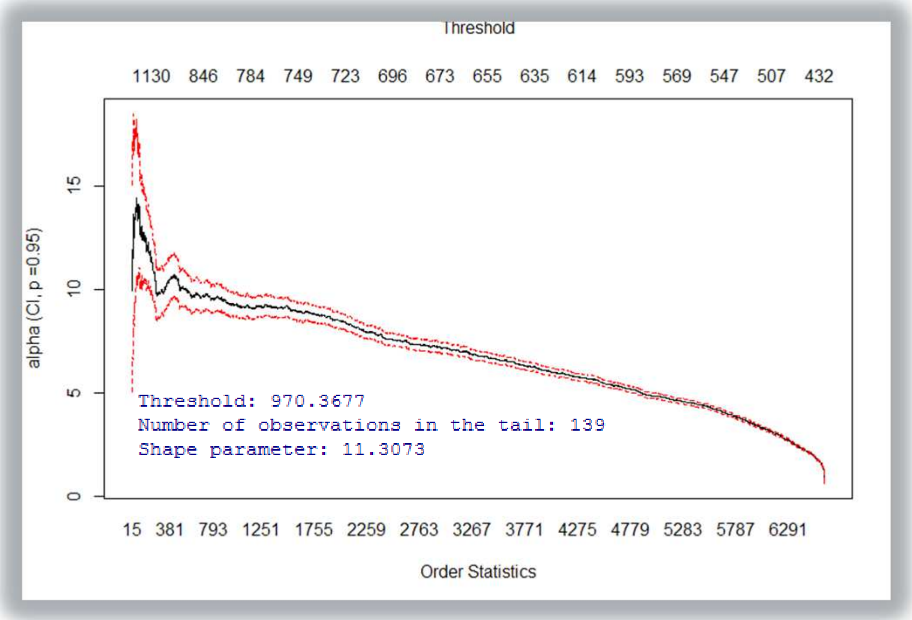
Quantile to quantile representation (qqplot)



Extreme value theory
Threshold determination

HOP!

Statistical techniques for threshold determination



- Hill estimator to evaluate the maximum likelyhood
- Mean Excess function
- « Winsorisation »
  Charles P. Winsor (1895–1951)
- Clipping technique

Extreme value theory

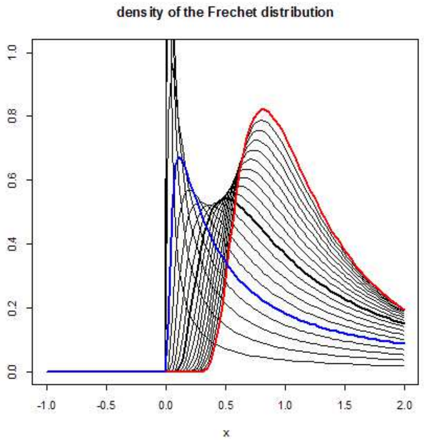
Generalized Extreme Value (GEV) distribution

The extreme values theory is used to study the limit distributions of extremas.

Extreme values can be modelled by the Generalized Extreme Value (GEV) distribution.

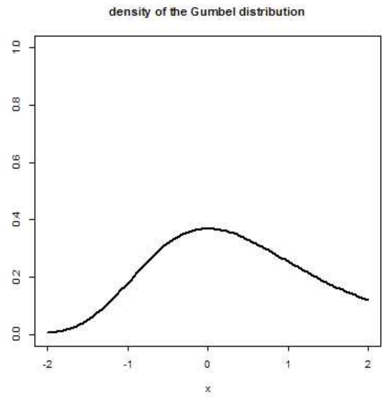
$$H(x) = \begin{cases} e^{-\left(1+\xi\frac{x-\mu}{\psi}-x\right)^{-\frac{1}{\xi}}} & if \xi \neq 0 \\ e^{-e^{-\left(\frac{x-\mu}{\psi}\right)}} & if \xi = 0 \end{cases}$$

Fréchet



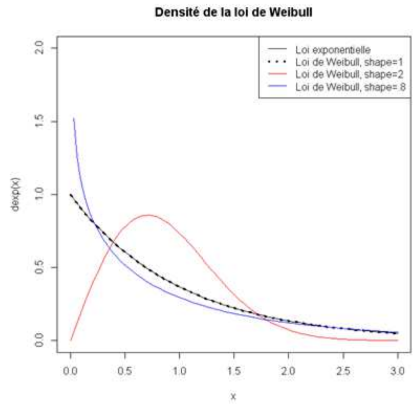
$\xi > 0$

Gumbel



$\xi = 0$

Weibull

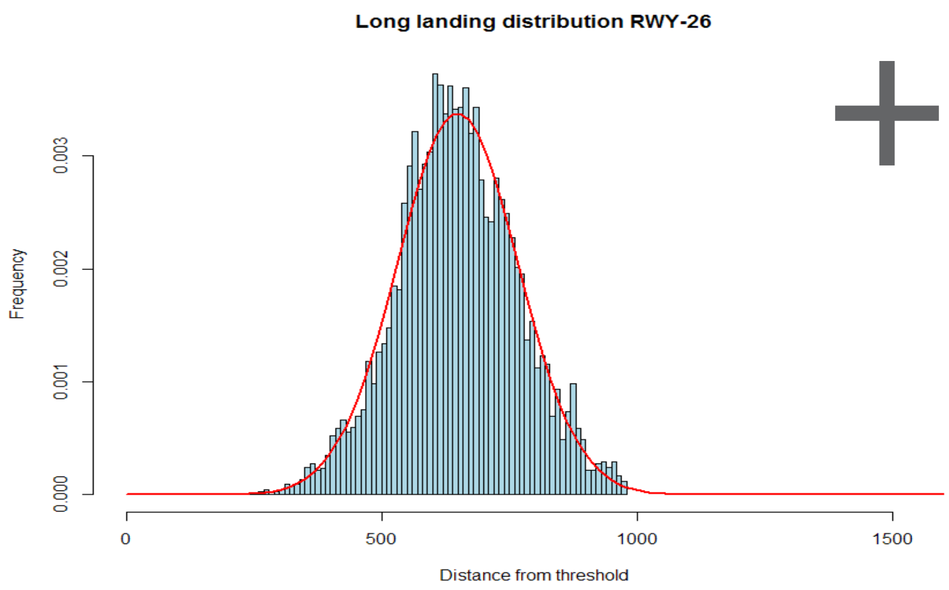


$\xi < 0$

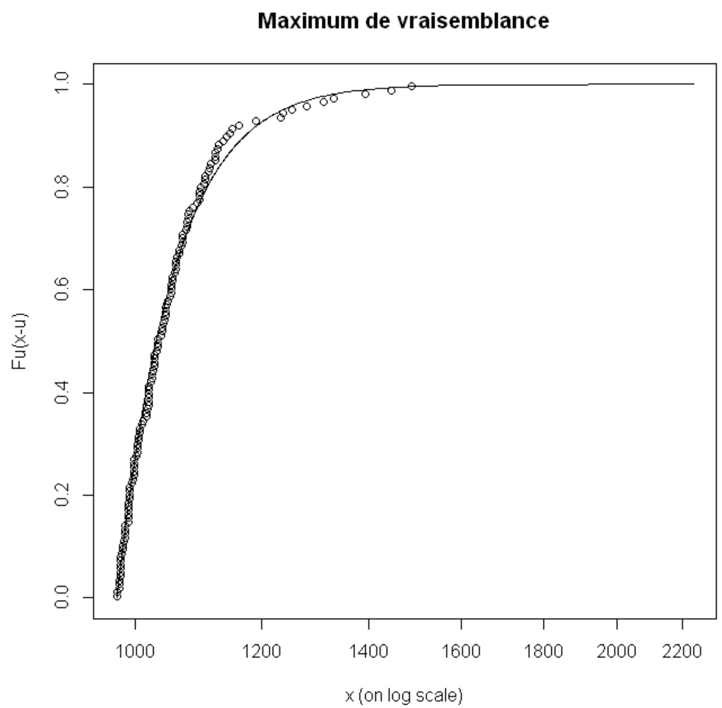
Extreme value theory

Modelisation

- RWY-26 without the exceedance (extreme values) is now correctly approximated by a Gaussian distribution :
  - mean : 649 m ; sd : 118 m
  - Excess kurtosis : 0,02 ;
  - Skewness = 0,02



- RWY-26 extreme values are approximated with a Generalized Extreme Value distribution.



HOP!

Risk models exist in other industries …

- In finance or in insurance : model of the « Value at Risk »,
- In finance, or in insurance, the VaR, is an estimation of the potential loss,
- With the concept of the VAR, a single figure can express the risk of a portfolio.

The results

Long landing RWY-26

Level of confidence of 95%, 99%, 99,5%, expectation of not exceeding the limit.

- Normal (Gaussian) VaR

$$VaR_\alpha = \mu - z_\alpha \sigma$$

Normal VaR		
95 %	99 %	99,5%
776 m	826 m	844 m

- Cornish Fisher VaR

$$VaR_{1-p} = \mu + \sigma[k_{1-p} + Sk/6(k_{1-p}^{2} - 1) + Ku/24(k_{1-p}^{3} - 3k_{1-p}) - Sk^2/36(2k_{1-p}^{3} - 5k_{1-p})]$$

Cornish Fisher VaR		
95 %	99 %	99,5%
785 m	873 m	1020 m

- Extreme VAR

Extreme VaR		
95 %	99 %	99,5%
903 m	1031 m	1090 m

The results

Long landing RWY-26

Safety Performance Indicator for RWY-26 :

→ Probability of landing distances over 900 m

Long landing VaR at 900 m		
Normal VaR	Cornish Fisher VaR	Extreme VaR
0.04 %	0.61 %	5.19 %

Summary

HOP!

- Risk management is the management of the unexpected events,

- Classic modelisations with the Gaussian distribution (mean/sd) can conduct to a severe underestimation of your risk,

- Be aware of your statistics,

- An « extreme » model is pertinent…

to highlight your …

And that’s it,

Thank you, for your attention.